These Combinatorics NOTES Belong to:___________

Date	Topic	Notes	Questions
1.	Chapter Summary	2,3	
2.	Fundamental Counting Principle	4-8	
3.	Permutations	9-13	
4.	Permutations	14-17	
5.	Combinations	18-22	
6.	Combinations	23-26	
7.	Pascal's triangle	27-30	
8.	Binomial Theorem	31-37	
9.	Binomial Theorem	38-42	
10.	Review		
11.	Review		
12.	TEST		

Please Read:

Some students find this chapter and the probabilities chapter hard. Their main complaint is that they are unsure about whether the question is a permutation question or a combination question. Find out what a permutation and combination is as soon as possible. It will make the ride much smoother. These concepts are defined on page 2.

IQ TEST:

Combination: How many ways can you choose 2 fingers from your left hand?

Permutation: How many ways can you tap the fingers of one hand on your desk?

Record any questions that you find challenging.

Combinatorics Summary Page #1

1. The Fundamental Counting Principle.

- If one item can be selected in X ways, and for every way a second item can be selected in y ways, then the two items can be selected in XY ways.

2. Factorial Notation! → 5! Is read 5 factorial.

1!	2!	3!	4!	5!	N!
1	$1\times2=2$	$1\times2\times3=6$	$1\times2\times3\times4=24$	$1\times2\times3\times4\times5=120$	$1\times2\times3\times\ldots\times(N-1)\times N$

Where is the **!** **button** on my calculator? Press MATH, choose C PROB, **select !**, press enter.

IMPORTANT DEFINITION→ **0!=1**

3. Permutations and Combinations

Permutation*: $_nP_r=\dfrac{n!}{(n-r)!}$	Combination*: $_nC_r=\dfrac{n!}{r!(n-r)!}$
Placement/order matters	**Placement/order does not matter**
❖ Races with 1st, 2nd, 3rd	❖ Races where top 3 advance
❖ Electing a president and vice president	❖ Electing 2 co-presidents
❖ Arranging books on a shelf	❖ Throwing books in a garbage can with a lid.
❖ Arranging fruit in glass bowl	❖ Place the fruit from the bowl in a blender
❖ Card games like Speed	❖ Card games like poker
Official Definition	**Official Definition**
❖ An ordered arrangement of distinct objects	❖ An unordered arrangement of distinct objects
❖ The number of permutations of n distinct objects taken r at a time	❖ The number of combinations of n distinct objects taken r at a time
Graphing Calculator Help	**Graphing Calculator Help**
❖ $_5P_2$	❖ $_5C_2$
❖ 5→ Math→Prob→ $_nP_r$→enter→2→enter	❖ 5→ Math→Prob→ $_nC_r$→enter→2→enter

* $n>0$, $r>0$, $n>r$ and both n and r are whole numbers.

4. Permutations Involving Identical Objects

The number of permutations of n objects of which there are a objects alike of one kind, b alike of another kind, c alike of another kind, and so on is: $\dfrac{n!}{a!b!c!\ldots}$

Combinatorics Summary Page #2

5. Pascal's Triangle

Compare ${}_nC_r$ in each row with the coefficients in the binomial expansions.

Pascal's Triangle	Row	Binomial	Expansion
${}_0C_0$	Row 1	$(x+y)^0$	**1**
${}_1C_0$ ${}_1C_1$	Row2	$(x+y)^1$	**1**x+**1**y
${}_2C_0$ ${}_2C_1$ ${}_2C_2$	Row3	$(x+y)^2$	**1**x^2+**2**xy+**1**y^2
${}_3C_0$ ${}_3C_1$ ${}_3C_2$ ${}_3C_3$	Row4	$(x+y)^3$	**1**x^3+**3**x^2y+**3**xy^2+**1**y^3
${}_4C_0$ ${}_4C_1$ ${}_4C_2$ ${}_4C_3$ ${}_4C_4$	Row5	$(x+y)^4$	**1**x^4+**4**x^3y+**6**x^2y^2+**4**xy^3+**1**y^4
${}_5C_0$ ${}_5C_1$ ${}_5C_2$ ${}_5C_3$ ${}_5C_4$ ${}_5C_5$	Row6	$(x+y)^5$	**1**x^5+**5**x^4y+**10**x^3y^2+**10**x^2y^3+**5**xy^4+**1**y^5

The r value of ${}_nC_r$ in each box is equal to the y exponent in each term.

6. The Binomial Theorem

Expand $(X+Y)^6$ using the Binomial Theorem

How many terms will there be?

- The coefficient of each term can be found using ${}_6C_{\underline{0}} \rightarrow {}_6C_{\underline{6}}$
- X exponents start at 6 and decrease by 1 until it reaches 0
- Y exponents start at <u>0</u> and increase by 1 until it reaches <u>6</u>
- Each term is the product of the above

Term 1		Term 2		Term 3		Term 4		Term 5		Term 6		Term 7	
${}_6C_0$	= 1	${}_6C_1$	= 6	${}_6C_2$	= 15	${}_6C_3$	= 20	${}_6C_4$	= 15	${}_6C_5$	= 6	${}_6C_6$	= 1
X^6	= X^6	X^5	= X^5	X^4	= X^4	X^3	= X^3	X^2	= X^2	X^1	= X^1	X^0	= 1
y^0	= 1	y^1	= y	y^2	= y^2	y^3	= y^3	y^4	= y^4	y^5	= y^5	y^6	= y^6
X^6		+ $6X^5y$		+ $15X^4y^2$		+ $20X^3y^3$		+ $15X^2y^4$		+ $6Xy^5$		+ y^6	

$$(X+Y)^6 = X^6 + 6X^5Y + 15X^4Y^2 + 20X^3Y^3 + 15X^2Y^4 + 6XY^5 + Y^6$$

7. Use the formula to find a specific term in an expansion

Find the (k+1)th term $t_{k+1} = {}_nC_k x^{n-k} y^k$

This formula is given on the provincial exam

8. Mixing all the ideas together→ see page 30-31
9. How do you do combinations questions with the words at least or at most?

Combinatorics Challenge Questions

(Challenge questions are designed to be completed in small groups and supported by the teacher.)

A. How many ways can you choose 2 fingers from your left hand?

B. How many ways can you tap the fingers of one hand on your desk?

C. How many phone 7 digit-phone-numbers can exist in a city starting with 598?

D. Theoretically speaking, how many phone numbers can exist in a ten-digit-BC-phone number with the area code 250?

E. Considering restrictions, how many numbers do you think are actually possible with area code beginning with 250?

Problem Questions: Students should be given ample time to try the "Problem" questions before they seek help or are given an explanation by their teacher.

Problem #1: Stu Dent just bought 2 hats, 3 shirts, and 2 pairs of pants. How many different outfits are possible?

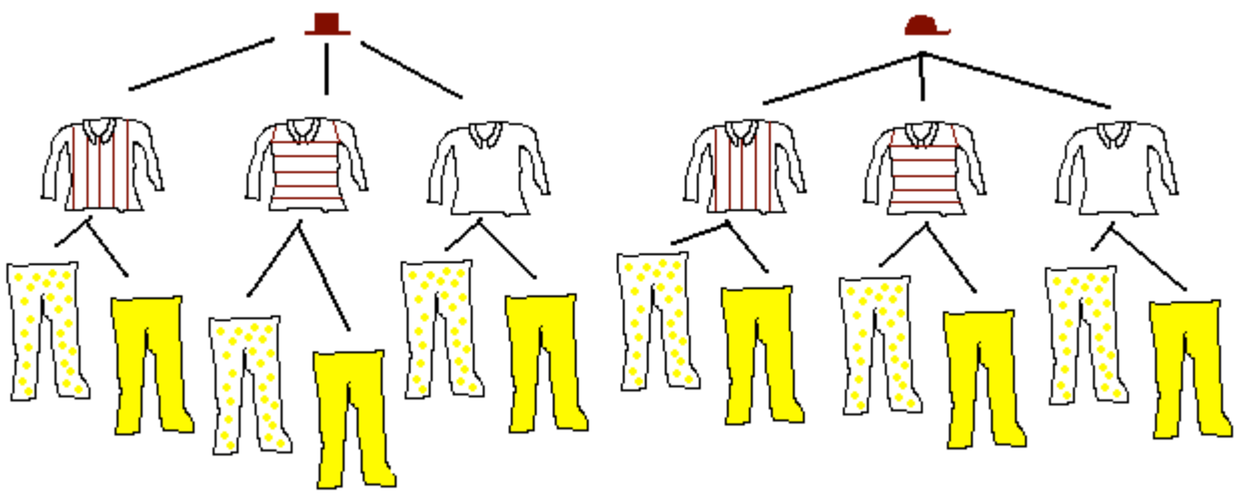
Fundamental Counting Principle

The Fundamental Counting Principle.

- If one item can be selected in X ways, and for every way a second item can be selected in y ways, then the two items can be selected in XY ways.
- See question 1 below.

1. Stu Dent just bought 2 hats, 3 shirts, and 2 pairs of pants. How many different outfits are possible?

A tree diagram can be used to help you see the different options.



There are 12 different options.

OR $2hats \times 3shirts \times 2pants = 12$ (The fundamental counting principle)

2. Sam's Deli features 3 kinds of bread. They have 4 different kinds of meat and 2 different kinds of cheese. How many different sandwiches can be made?

Draw a tree diagram to solve.	Use a formula.

24

2 digit numbers (09 is not a 2 digit number)

3. How many 2 digit numbers are possible?

Solution:
- How many options for the 1st digit are there?
- How many options are there for the 2nd digit?

4. How many odd 2 digit numbers are possible?

5. How many 2 digit numbers can be created so that no number repeats?

3 digit numbers (029 is not a 3 digit number)

6. How many 3 digit numbers are possible?

1st digit options X 2nd digit options X 3rd digit options

7. How many odd 3 digit numbers are possible?

8. How many 3 digit numbers exist where no # repeats?

3 digit codes

9. How many different 3 digit codes can be made if the first digit has to be even, the second digit must be a vowel (y is not a vowel) and the last digit must be an x,y or z.

10. How many different 3 digit codes can be made if the first digit has to be between 5 and 9 inclusive, the second digit must be a single digit # and the last digit must be an M,A,T or H.

11. How many different 3 digit codes can be made if the first digit has to be a letter, the second digit must be an x or y and the last digit must be a number between 1-5 inclusive.

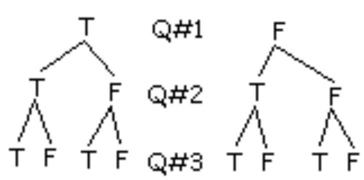
Problem #2: A true-false test has 3 questions.

- How many answer keys are possible? (Use a list and a tree diagram to answer this question)
- What is the probability of guessing every question right?

Draw a tree diagram or use a chart to organize your ideas.

12. A true-false test has 3 questions. How many answer keys are possible?

Solution:
Question #1→2 options
Question #2→2 options
Question #3→2 options



The # of answer keys can be found by:
$2 \times 2 \times 2 = 2^3 = 8$

13. What is the probability of guessing every question right?

Solution:
$p(100\%) = \frac{1}{8} or 0.125$

14. A true-false test has 5 questions. How many answer keys are possible?

32

15. P(100%)=

3.125%

16. A true-false test has 8 questions. How many answer keys are possible?

17. P(100%)=

18. A 5 question multiple choice test has options a, b, c, & d. How many answer keys are possible?

19. P(100%)=

20. A 10 question multiple choice test has options a, b, c, d & e. How many answer keys are possible?

21. P(100%)=

Problem #3: The final score of a soccer game is 4-2. How many scores are possible at the start of the second half?

15

22. How many different ice cream sundaes can be made from 5 choices of ice cream and 3 choices of topping if only one flavor and one topping is selected for each sundae.

15

23. How many different punches can be made from 3 choices of pop and 7 choices of juice if only one pop and one juice is selected for drink.

24. The final score of a hockey game was 3-2. How many scores are possible after the 1st period?

Hint(List all possible scores for each team)

Problem #4: How many unique four letter arrangements can be made from the letters MATH?

- Before you begin, estimate the number of possibilities→______
- Now find a strategy to find the exact number of solutions.

Factorial Notation

Answer from the previous page. (MATH has 24 unique arrangements)

How many ways can you rearrange the following letters?

25. TSPRAY
Solution:
6 letters to pick to be 1st
5 letters to pick to be 2nd
4 letters left to pick the 3rd
3 letters left to pick the 4th
2 letters left to pick the 5th
1 letter left to pick the 6th
$6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$

26. LUVS

27. HIS

28. NOTES

Introduction: Factorial notation. 4! Is read four factorial. Factorial notation is helpful in solving many types of problems.

Problem #5: Determine what ! means by finding the pattern.

Evaluate 1!.	Evaluate 2!.	Evaluate 3!.	Evaluate 4!.	Evaluate 5!.
(Graphing Calculator) Press MATH, choose C PROB, **select !**, press enter.				

What does the ! mean in math?

Problem #6: Do not evaluate. What does 4! mean?

29. How many ways can 7 people finish a race?

30. How many ways can a doctor arrange 8 appointments?

31. How many ways can 9 people place in a race?

32. How many ways can 6 different books be placed on a shelf?

The use of pictures is often helpful in visualizing the problem. Have fun.

33. 5 kids are crossing the street holding hands in a straight line. If their parents have to be on each end, how many arrangements of the family are possible?

*

34. There are 10 different books on a bookshelf and 2 different bookends. How many different arrangements are possible?

35. There are 3 boys and 2 girls. How many ways can they arrange themselves in a straight line if genders must alternate?

*Is your answer only half of the correct answer? Did you draw a picture?

Problem #7: Seven students were nominated for best overall math hairstyle. How many different ways can 1st, 2nd, 3rd be awarded?

Permutations Involving Different Objects

Definition.

Permutation: ${}_nP_r = \dfrac{n!}{(n-r)!}$

Definition:

- An ordered arrangement of distinct objects
- The number of permutations of n distinct objects taken r at a time

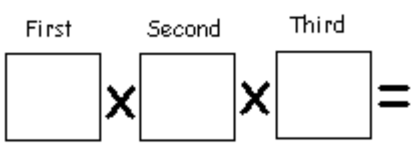
Example. How many ways can Blair, Matt & Dave finish in a race? (BMD, BDM, ,MBD, MDB, DMB & DBM)

Placement/order matters

- Races with 1st,2nd,3rd
- Electing a president and vice president
- Arranging books on a shelf
- Arranging fruit in glass bowl
- Card games like Speed

36. Seven students were nominated for best overall math hairstyle. How many different ways can 1st, 2nd, 3rd be awarded?

How many people can come 1st?
How many people can come 2nd?
How many people can come 3rd?



Select 3 items from 7→ ${}_nP_r = \dfrac{n!}{(n-r)!}$

${}_7P_3 = \dfrac{7!}{(7-3)!} \rightarrow {}_7P_3 = \dfrac{7!}{4!} \rightarrow {}_7P_3 = \dfrac{7 \times 6 \times 5 \times 4!}{4!}$

${}_7P_3 = 7 \times 6 \times 5 = 210$

Problem #8: Evaluate ${}_{10}P_2 =$ ${}_5P_3 =$ ${}_6P_6 =$

90,60,720

37. There are 7 people in a race. How many different ways are there to assign 1^{st}, 2^{nd} and 3^{rd}.

Placement matters, use the formula

$${}_nP_r = \frac{n!}{(n-r)!}$$

Fill out formula

$${}_7P_3 = \frac{7!}{(7-3)!} = \frac{7!}{4!} = 210$$

There are 7 to pick from and 3 are chosen.

210

38. There are 10 people in a race. How many different ways are there to assign 1^{st}, 2^{nd} and 3^{rd}.

720

39. There are 100 people in a race. How many different ways are there to assign 1^{st}, 2^{nd} and 3^{rd}.

Problem #9: Use your graphing calculator to evaluate ${}_{10}P_2 =$

Press 10→ Math→Prob→ ${}_nP_r$→enter→2→enter

Determine the answer.

40. There are 20 songs on a CD. If only 4 songs can be played during class, how many different music sets are possible?

41. There are 50 songs on a CD. If only 3 songs can be played during class, how many different music sets are possible?

42. The top three prizes in a raffle are an Ipod, a cd player and movies tickets. How many ways can the prizes be awarded if 100 people buy tickets?

Problem #10: How would you represent the number just before n? (The answer is not m)

- What is the number just before (n+2)?
- What is the number just before (n-1)?

n-1,n+1,n-2

Problem #11: Evaluate $\frac{1000!}{998!}$.

Problem #12: What is bigger n or (n-2)? Simplify $\frac{n!}{(n-2)!}$.

$n^2 - n$

Simplify and write the following without factorial notation.

43. Evaluate. ${}_9P_2 =$

44. Evaluate. ${}_{50}P_3 =$

45. Evaluate. ${}_{100}P_2 =$

46. Evaluate. ${}_nP_3 =$

47. Evaluate. ${}_{n+1}P_1 =$

48. Evaluate. $\frac{11!}{9!} =$

49. Evaluate. $\frac{9!}{6!} =$

50. Evaluate. $\frac{1000!}{998!} =$

1000! Is too big for your calculator to compute. Use the definition of factorial and reduce.

$$\frac{1000 \times 999 \times 998!}{998!}$$

Reduce.

$1000 \times 999 = 999000$

51. Evaluate. $\frac{102!}{99! \times 101} =$

Problem #13: ${}_nP_2 = 110$ Determine the value of n using any method. Show your work.

Simplify.

52. $\dfrac{n!}{(n-2)!} =$

Solution:

$\dfrac{n!}{(n-2)!} = \dfrac{n(n-1)(n-2)!}{(n-2)!}$

Reduce

$\dfrac{n!}{(n-2)!} = n(n-1)$ or $n^2 - n$

53. $\dfrac{(n+2)!}{(n)!} =$

54. $\dfrac{n(n-2)!}{(n-3)!} =$

55. $\dfrac{(n-1)!}{4(n-3)!} =$

Find n.

56. ${}_nP_2 = 110$ Find n.

Fill out formula

${}_nP_2 = \dfrac{n!}{(n-2)!} = 110$

Manipulate to remove factorial

$110 = \dfrac{n(n-1)(n-2)!}{(n-2)!}$

Reduce

$110 = n(n-1)$

$110 = n^2 - n$

$0 = n^2 - n - 110$

Factor and solve

$0 = (n-11)(n+10)$

n=11 or n=-10(reject (-10))

57. ${}_nP_2 = 20$ Find n.

58. ${}_nP_2 = 56$ Find n.

Problem #14: ${}_9P_n = 72$ Determine the value of n using any method. Show your work.

59. Find n. ${}_{9}P_{n} = 72$

$${}_{9}P_{n} = \frac{9!}{(9-n)!} = 72$$

$$72 = \frac{9!}{(9-n)!}$$

Cross multiply

$$(9-n)! = \frac{9!}{72}$$

Reduce

$$(9-n)! = \frac{9\times 8\times 7!}{9\times 8}$$

$$(9-n)! = 7!$$

so 9-n=7→n=2

2

60. Find n. ${}_{7}P_{n} = 42$

61. Find n. ${}_{10}P_{n} = 90$

62. A soccer coach has 10 players to choose from. He must decide 5 players to take penalty shots and then decide who will shoot 1st,2nd,3rd,... Determine the number of different arrangements the coach has to choose from.

63. A soccer coach must choose 3 out of 8 players to kick the shoot out. Assuming the coach must designate the order of the 3 players; determine the number of different arrangements the coach has to choose from.

64. The final score of a baseball game was 7-4. How many scores are possible after the 5th inning?

65. There are 3 boys and 3 girls. How many ways can they arrange themselves in a straight line if genders must alternate?

66. A 6 question multiple choice test has options a, b, c, d and e. How many answers keys are possible?

67. There are 50 songs on a CD. If only 3 songs can be played during class, how many different music sets are possible?

68. How many different ice cream sundaes can be made from 6 choices of ice cream and 4 choices of topping, if only one flavor and one topping is selected for each sundae.

69. The final score of a soccer game was 5-3. How many scores are possible after 20 minutes of play?

Permutations Involving Identical Objects→ $\frac{n!}{a!b!c!...}$

Problem #15:

- How many unique 4-digit numbers can be created with the numbers 1,2,3,4?

1234	2134	3124	4123
1243	2143	3142	4132
1324	2314	3214	4213
1342	2341	3241	4231
1423	2413	3412	4312
1432	2431	3421	4321

$4! = 24$

- How many unique 4-digit numbers can be created with the numbers 1,1,3,4?

Use a table to show the answer.

Formula→

How many duplicates are created by having two ones?

- How many unique 4-digit numbers can be created with the numbers 1,1,1,4?

Use a table to show the answer.

Formula→

How many duplicates are created by having three ones?

- How many unique 4-digit numbers can be created with the numbers 1,1,1,1?

Use a table to show the answer.

Formula→

How many duplicates are created by having four ones?

Permutations Involving Identical Objects→ $\frac{n!}{a!b!c!...}$

The number of permutations of n objects of which there are a objects alike of one kind, b alike of another kind, c alike of another kind, and so on is: $\frac{n!}{a!b!c!...}$

Problem #16: How many different permutations can be made from the letters MEYE?

How many different permutations can be made from the following letters?

70. MEYE

Solution:

$$\frac{4!}{1!2!1!}=12$$

12

71. STUDAUNTS

45360

72. LLLUVVV

73. THEEEZ

74. NOOOOTCC

75. Given: A 7 question true false test answer key has 5T and 2F. How many different answer keys are possible?

21

76. Given: 10 question multiple-choice. Answer key has 5 A's, 2B's, 2C's and 1D. How many answer keys are possible?

77. Given: 15 question multiple-choice. Answer key has 3 A's, 10B's, 1C's and 1D. How many answer keys are possible?

78. Nine buttons differ by color only. 3 are green, 3 are red and 3 are orange. How many different ways can they be arranged vertically on a shirt?

79. How many unique arrangements of the number 123412341234 are possible?

80. Ten buttons differ by color only. 3 are green, 3 are red and 4 are orange. How many different ways can they be arranged vertically on a shirt?

Combinations

Developing the concept of a Combination

Adam, Bryan, Colin & David will be running in a four person race. The following is a list of the possible race outcomes.

A,B,C,D	A,B,D,C	A,C,D,B	B,C,D,A
A,C,B,D	A,D,B,C	A,D,C,B	B,D,C,A
B,A,C,D	B,A,D,C	C,A,D,B	C,B,D,A
B,C,A,D	B,D,A,C	C,D,A,B	C,D,B,A
C,A,B,D	D,A,B,C	D,A,C,B	D,B,C,A
C,B,A,D	D,B,A,C	D,C,A,B	D,C,B,A

81. In a four person race, how many ways can 1st, 2nd and 3rd place be awarded?

Solution:

This is a permutation of 4 runners taken 3 at a time.

$${}_4P_3 = \frac{4!}{(4-3)!} = \frac{4!}{1!}$$

$${}_4P_3 = 4 \times 3 \times 2 = 24$$

The solution is all the arrangements above!

Does placement matter?

YES

82. Adam, Bryan, Colin & David just found out that the top 3 finishers will make the national team. How many different national teams can be formed?

Solution:

Since the top 3 runners make the national team, the focus is not so much in winning the race as it is in being one of the top 3. Therefore A,B,C,D & A,C,B,D will create the same Olympic team. *Placement in the top 3 does not matter.*

We can solve this question by thinking about the race in terms of how many will Qualify and how many will be Cut. We will not think of them in terms of 1st,2nd,3rd&4th but rather Q,Q,Q,C.

Remember identical permutations→ $\frac{4!}{3!1!} = 4$

Each column above represents a single solution.

Does placement matter to first 3 runners?

NO

Permutation	Combination
${}_nP_r = \frac{n!}{(n-r)!}$	${}_nC_r = \frac{n!}{r!(n-r)!}$ → ${}_4C_3 = \frac{4!}{3!(4-3)!} = \frac{4!}{3!1!} = 4$

PERMUTATION VERSES COMBINATION

Permutation: $_nP_r = \frac{n!}{(n-r)!}$	Combination: $_nC_r = \frac{n!}{r!(n-r)!}$
Placement/order matters	**Placement/order does not matter**
❖ Races with 1^{st}, 2^{nd}, 3^{rd} ❖ Electing a president and vice president ❖ Arranging books on a shelf ❖ Arranging fruit in glass bowl ❖ Card games like Speed	❖ Races where top 3 advance ❖ Electing 2 co-presidents ❖ Throwing books in a garbage can with a lid. ❖ Place the fruit from the bowl in a blender ❖ Card games like poker
Official Definition	**Official Definition**
❖ An ordered arrangement of distinct objects. ❖ The number of permutations of n distinct objects taken r at a time.	❖ An unordered arrangement of distinct objects. ❖ The number of combinations of n distinct objects taken r at a time.
Graphing Calculator Help	**Graphing Calculator Help**
❖ $_5P_2$ ❖ 5→ Math→Prob→ $_nP_r$→enter→2→enter	❖ $_5C_2$ ❖ 5→ Math→Prob→ $_nC_r$→enter→2→enter

Problem #17: A bowling team is made up of 10 kids. How many different groups of 4 can be sent to the city championships?

State what kind of question it is, permutation or combination and then find the answer.

83. A bowling team is made up of 10 kids. How many different groups of 4 can be sent to the city championships? (P or C)

Solution

$$_{10}C_4 = \frac{10!}{4!6!}$$

84. A golf team is made up of 8 kids. How many different groups of 3 can be sent to the provincial championships? (P or C)

85. How many ways can a president, vice president and a twit be selected from the 30 students? (P or C)

86. How many ways can 1^{st}, 2^{nd} and 3^{rd}, be awarded to the 10 bowlers? (P or C)

87. How many ways can 1^{st}, 2^{nd} and 3^{rd}, be awarded to the 8 golfers? (P or C)

88. A class is made up of 30 kids. How many different committees of 3 can be formed? (P or C)

Problem #18: A bowling team is made up of 6 boys and 4 girls. How many different groups of 4 can be sent to the city championships if 2 boys and 2 girls have to go?

90

Problem #19: There are 5 boys and 6 girls in a class. How many ways can you select a king, a queen and a twit?

270

Solve the following problems and state whether it is a permutation or combination.

A bowling team is made up of 6 boys and 4 girls.	A golf team is made up of 5 boys and 3 girls.	A class is made up of 10 boys and 20 girls.
89. How many different groups of 4 can be sent to the city championships if 2 boys and 2 girls have to go? (P or C) Solution: Choose 2 boys from 6→ ${}_6C_2 = 15$ different pairs of boys Choose 2 girls from 4→ ${}_4C_2 = 6$ different pairs of girls Multiply the results ${}_6C_2 \times {}_4C_2 = 15 \times 6 =$ 90	90. How many different groups of 3 can be sent to the provincial championships if 2 boys and 1 girl have to go? (P or C)	91. How many committees of 3 can be formed if 1 boy and 2 girls have to be elected? (P or C)

92. There are 5 boys and 6 girls in a class. How many ways can you select a king, a queen and a twit?

Solution:

Choose 1 king from 5 boys→ ${}_5C_1$

Choose 1 queen from 6 girls→ ${}_6C_1$

Choose 1 twit from those not picked→ ${}_9C_1$

Multiply results ${}_5C_1 \times {}_6C_1 \times {}_9C_1 =$

270

93. There are 7 boys and 3 girls in a class. How many ways can you select best guy hair, best girl hair and worst hair day?

94. A deck of cards has 52 cards. How many different 5 card hands can be made up of exactly 2 kings, exactly 2 queens and 1 other card?

95. A deck of cards has 52 cards. How many different 5 card hands can be made up of exactly 1 king, exactly 2 queens and 2 other cards?

96. A deck of cards has 52 cards. How many different 5 card hands can be made up of exactly 2 spades, exactly 1 heart and 2 other cards?

97. A deck of cards has 52 cards. How many different 5 card hands can be made up of exactly 3 face cards, exactly 1 ace and 1 other card?

Problem #20: A committee is to be made up of 5 people.

- What are the possible compositions if there must be at least 4 boys?
- What are the possible compositions if there must be at most 2 girls?

Problem #21: A 5 person committee has been organized to deal with overhead pen recycling. 7 boys and 10 girls have expressed interest. How many committees are possible if there must be at least 4 girls?

(4b +1g or 5b), (3b+2g, 4b+1g or 5b), 1722

Solving Combination problems with the words exactly, at least and at most.

A committee of five will be chosen from 10 boys and 8 girls.

98. What are the possible gender compositions if the committee must have exactly 4 girls?

99. What are the possible gender compositions if the committee must have at least 4 girls?

See 101 for help

100. What are the possible gender compositions if the committee must have at most 2 girls?

See 104 for help

A 5 person committee has been organized to deal with overhead pen recycling. 7 boys and 10 girls have expressed interest.

101. How many committees are possible if there must be at least 4 girls?

At least 4 girls means:

1 boy, 4 girls→ ${}_7C_1 \times {}_{10}C_4 =$

0 boys, 5 girls→ ${}_7C_0 \times {}_{10}C_5 =$

Add the two rows

1722

102. How many committees are possible if there must be at least 3 guys?

103. How many committees are possible if there must be at least 2 girls?

A 5 person committee has been organized to deal with overhead pen recycling. 7 boys and 10 girls have expressed interest.

104. How many committees are possible if there must be at most 2 guys?

At most 2 guys means 0,1,2

0b,5g→ ${}_7C_0 \times {}_{10}C_5 =$

1b,4g→ ${}_7C_1 \times {}_{10}C_4 =$

2b,3g→ ${}_7C_2 \times {}_{10}C_3 =$

Add the three rows

4242

105. How many committees are possible if there must be at most 2 girls?

106. How many committees are possible if there must be at most 3 guys?

107. A warehouse contains 8 different cars, 4 different SUVs and 3 trucks. If you were allowed to take any 4 you wanted, how many different possibilities are there?

108. A warehouse contains 6 different cars, 5 different SUVs and 4 trucks. How many possibilities are there if exactly 2 of your 4 choices have to be cars?

109. A warehouse contains 5 different cars, 5 different SUVs and 2 trucks. If you are allowed to take any 2 vehicles, how many different possibilities are there?

110. A warehouse contains 8 different cars, 4 different SUVs and 3 trucks. How many possibilities are there if exactly one of your 4 choices has to be a truck?

111. A warehouse contains 6 different cars, 5 different SUVs and 4 trucks. If you are allowed to take any four vehicles, how many different possibilities are there?

112. A warehouse contains 5 different cars, 5 different SUVs and 2 trucks. How many possibilities are there if both of your choices have to be SUVs?

Problem #22: ${}_nC_2 = 55$ Find n.

113. ${}_nC_2 = 55$ Find n.

Solution:

$${}_nC_2 = \frac{n!}{2!(n-2)!} = 55$$

Rearrange

$$\frac{n(n-1)(n-2)!}{2!(n-2)!} = 55$$

Reduce

$$\frac{n(n-1)}{2!} = 55$$

$$n(n-1) = 110$$

$$n^2 - n - 110 = 0$$

Solve by factoring→11,(reject -10)

11

114. ${}_nC_2 = 10$ Find n.

115. ${}_nC_2 = 28$ Find n.

Write an expression for n.

116. ${}_nC_2 =$

117. ${}_{n-1}C_2 =$

118. ${}_{n+1}C_2 =$

119. $\frac{{}_{n+3}C_3}{n+3} =$

Problem #23: How many unique 5 card hands can be dealt from a deck of 52 cards?

Problem #24: How many unique 5 card hands can be dealt from a deck of 52 cards if there needs to be exactly 1 five, 1 seven and 3 other cards?

Color	Suit	Non-Face Cards										Face Cards		
Red	Hearts	A	2	3	4	5	6	7	3	4	10	J	Q	K
Red	Diamonds	A	2	3	4	5	6	7	3	4	10	J	Q	K
Black	Clubs	A	2	3	4	5	6	7	3	4	10	J	Q	K
Black	Spades	A	2	3	4	5	6	7	3	4	10	J	Q	K

Find the number of 5 card hands made up of:	Formula	Number
Any cards		120.
Spades only		121.
Hearts only		122.
Red cards only		123.
2 spades, 3 hearts		124.
3 red, 2 black		125.
2 red,2 spades, 1 club		126.
2 fours, 3 fives		127.
4 kings, 1 ace		128.
1 five, 1 seven, 3 other cards→	${}_4C_1 x\ {}_4C_1 x\ {}_{44}C_3$	129.
4 "5"s→choose 1, 4 "7"s→ choose 1, 44 cards that are not 5 or 7 choose 3		

Problem #25: Billy wants to tip a waitress at Moxies. He has five denominations, a nickel, a dime, a quarter, a loonie & a toonie. Assuming he tips, how many different tip amounts are possible if he leaves at least one denomination.

See 136 for help! Don't look to soon!

From a deck of 52 cards, how many different 5 card hands can be formed in each case?

130. Exactly 2 fours.

131. Exactly 3 face cards.

132. Exactly 3 red cards.

133. At least 2 fours.

134. At least 3 red cards.

135. At most 3 red cards.

Billy wants to tip a waitress at Moxies. He has five denominations, \$1, \$2, \$5, \$10& \$20. Assuming he tips, how many different tip amounts are possible if:

136. 1,2,3,4 or 5 denominations are left?

Solution

Another way of saying this would be, he leaves at least 1 bill.

1→5 choose 1→ ${}_5C_1=5$

2→5 choose 2→ ${}_5C_2=10$

3→5 choose 3→ ${}_5C_3=10$

4→5 choose 4→ ${}_5C_4=5$

5→5 choose 5→ ${}_5C_5=1$

31

137. 1,2 or 3 denominations are left?

138. 4 or 5 denominations are left?

Pascal's Triangle

Determine the pattern. Fill out the first 9 rows of Pascal's triangle.

Row 1	1						
Row 2	1	1					
Row3	1	2	1				
Row 4	1	3	3	1			
Row 5							
Row 6							
Row 7							
Row 8							
Row 9							

There are many patterns in this triangle. Name at least 7.

Pascal's Triangle

Row 1	1
Row 2	1 1
Row3	1 2 1
Row 4	1 3 3 1
Row 5	1 4 6 4 1
Row 6	1 5 10 10 5 1
Row 7	1 6 15 20 15 6 1

What generalizations could we make about this triangle?

139. The number of terms in each row is equal to:

140. The outer boxes are always equal to:

141. The left half of the triangle is equal to:

142. Not including row 1, each box is the sum of:

Compare Pascal's triangle with the coefficients in the binomial expansions			
1	Row 1	$(x+y)^0$	**1**
1 1	Row 2	$(x+y)^1$	$\mathbf{1}x+\mathbf{1}y$
1 2 1	Row 3	$(x+y)^2$	$\mathbf{1}x^2+\mathbf{2}xy+\mathbf{1}y^2$
1 3 3 1	Row 4	$(x+y)^3$	$\mathbf{1}x^3+\mathbf{3}x^2y+\mathbf{3}xy^2+\mathbf{1}y^3$
1 4 6 4 1	Row 5	$(x+y)^4$	$\mathbf{1}x^4+\mathbf{4}x^3y+\mathbf{6}x^2y^2+\mathbf{4}xy^3+\mathbf{1}y^4$
1 5 10 10 5 1	Row 6	$(x+y)^5$	$\mathbf{1}x^5+\mathbf{5}x^4y+\mathbf{10}x^3y^2+\mathbf{10}x^2y^3+\mathbf{5}xy^4+\mathbf{1}y^5$

Use your calculator to fill out the $_nC_r$.

Row 1	$_0C_0=$	$(x+y)^0$
Row 2	$_1C_0=$ $_1C_1=$	$(x+y)^1$
Row3	$_2C_0=$ $_2C_1=$ $_2C_2=$	$(x+y)^2$
Row 4	$_3C_0=$ $_3C_1=$ $_3C_2=$ $_3C_3=$	$(x+y)^3$
Row 5	$_4C_0=$ $_4C_1=$ $_4C_2=$ $_4C_3=$ $_4C_4=$	$(x+y)^4$
Row 6	$_5C_0=$ $_5C_1=$ $_5C_2=$ $_5C_3=$ $_5C_4=$ $_5C_5=$	$(x+y)^5$
Row 7	$_6C_0=$ $_6C_1=$ $_6C_2=$ $_6C_3=$ $_6C_4=$ $_6C_5=$ $_6C_6=$	$(x+y)^6$
••••	• • • • • • • •	
Nth	$_{n-1}C_0$ $_{n-1}C_1$ $_{n-1}C_2$ • • • $_{n-1}C_{n-2}$ $_{n-1}C_{n-1}$ $_{n-1}C_n$	$(x+y)^{n-1}$

Symmetric Pattern (Mirror)	Recursive Pattern (Adding 2 above)
$_5C_2 = {_5C_3}$ =10	$_5C_2 = {_4C_1} + {_4C_2}$ =10
$_nC_r = {_nC_{(n-r)}}$	$_nC_r = {_{(n-1)}C_{(r-1)}} + {_{(n-1)}C_r}$

*Find an equivalent $_nC_r$. The symmetric pattern.

143. $_6C_2=$	144. $_{17}C_8=$	145. $_{100}C_{81}=$	146. $_nC_{10}=$	147. $_{n+10}C_{10}=$	148. $_{ab}C_c=$

*Find the $_nC_r$ that is the sum of the following. This is the recursive pattern.

149. $_5C_1 + {_5C_2}$	150. $_{44}C_{19} + {_{44}C_{20}}$	151. $_{a-1}C_{b-1} + {_{a-1}C_b}$	152. $_aC_b + {_aC_{b+1}}$
153. $_{a+6}C_{b+1} + {_{a+6}C_{b+2}}$	154. $_{2a+1}C_{b-5} + {_{2a+1}C_{b-4}}$	155. $_{a-x}C_{by-1} + {_{a-x}C_{by}}$	156. $_{Iluv-1}C_{math12-1} + {_{Iluv-1}C_{math12}}$

Find $_nC_r$ that satisfies the following.

157. 6th row, 4th position	158. 61st row, 1st position	159. 8th row, 2nd position	160. 101st row, 99th position	161. Ath row, 5th position	162. (d+3)th row, dth position

*Best way to solve these is to draw the first few rows and then generalize. (Easier when you can see it)

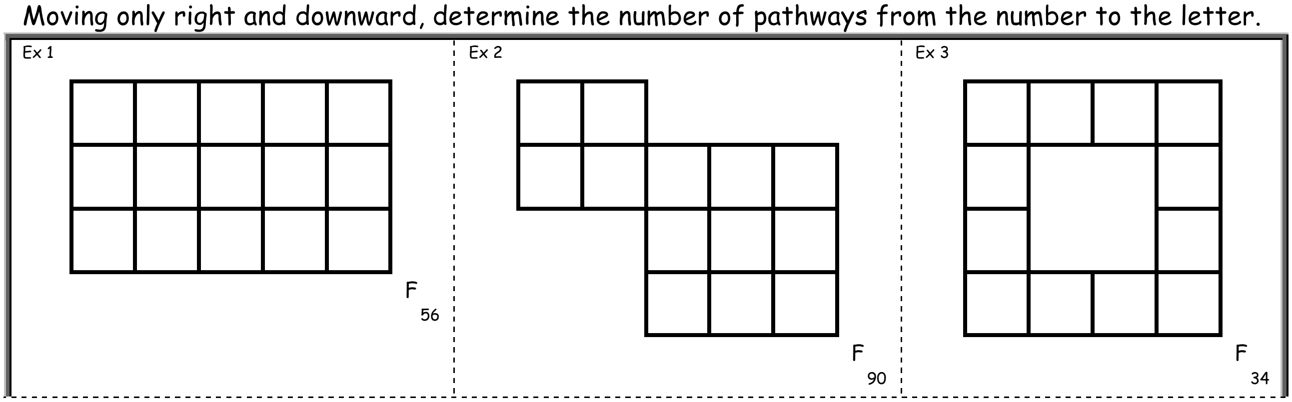
Problem #26: Draw a square. Place an a and a b at opposite corners. How many ways are there to get from a to b?

Problem #27: There are two question here. Draw a two by two grid and a 1 by 4 grid.   Place an a and a b at opposite corners of each grid. How many ways are there to get from a to b in each grid?

Problem #28: Draw a two by three grid. Place an a and a b at opposite corners. How many ways are there to get from a to b?

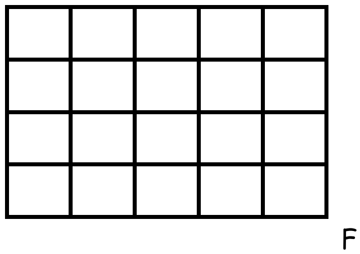
Describe two ways of solving the above problems.

Moving only right and downward, determine the number of pathways from the number to the letter.



Moving only right and downward, determine the number of pathways from the number to the letter.

163.



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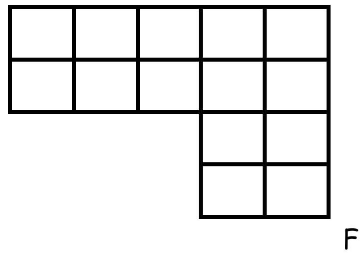

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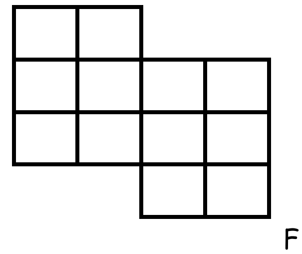

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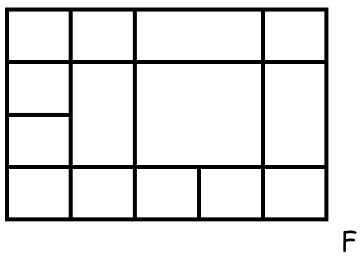

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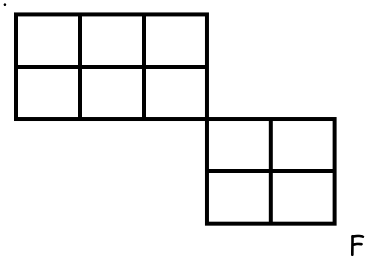

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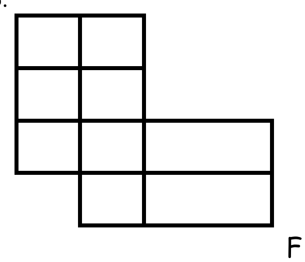

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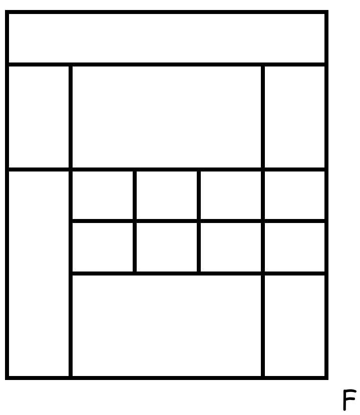

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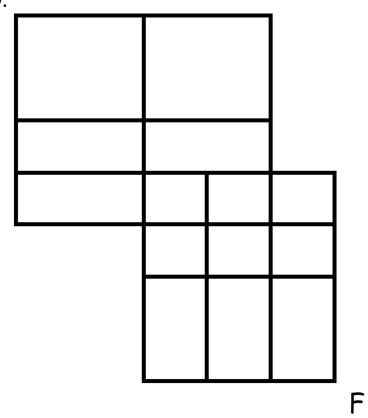

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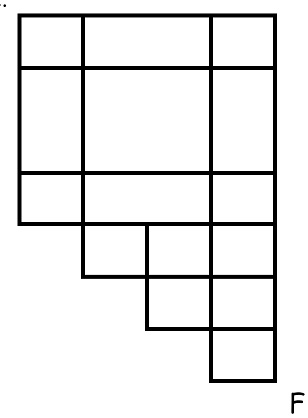
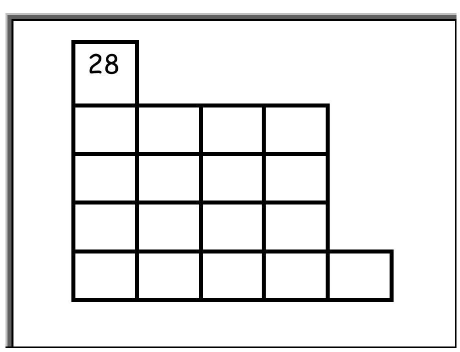
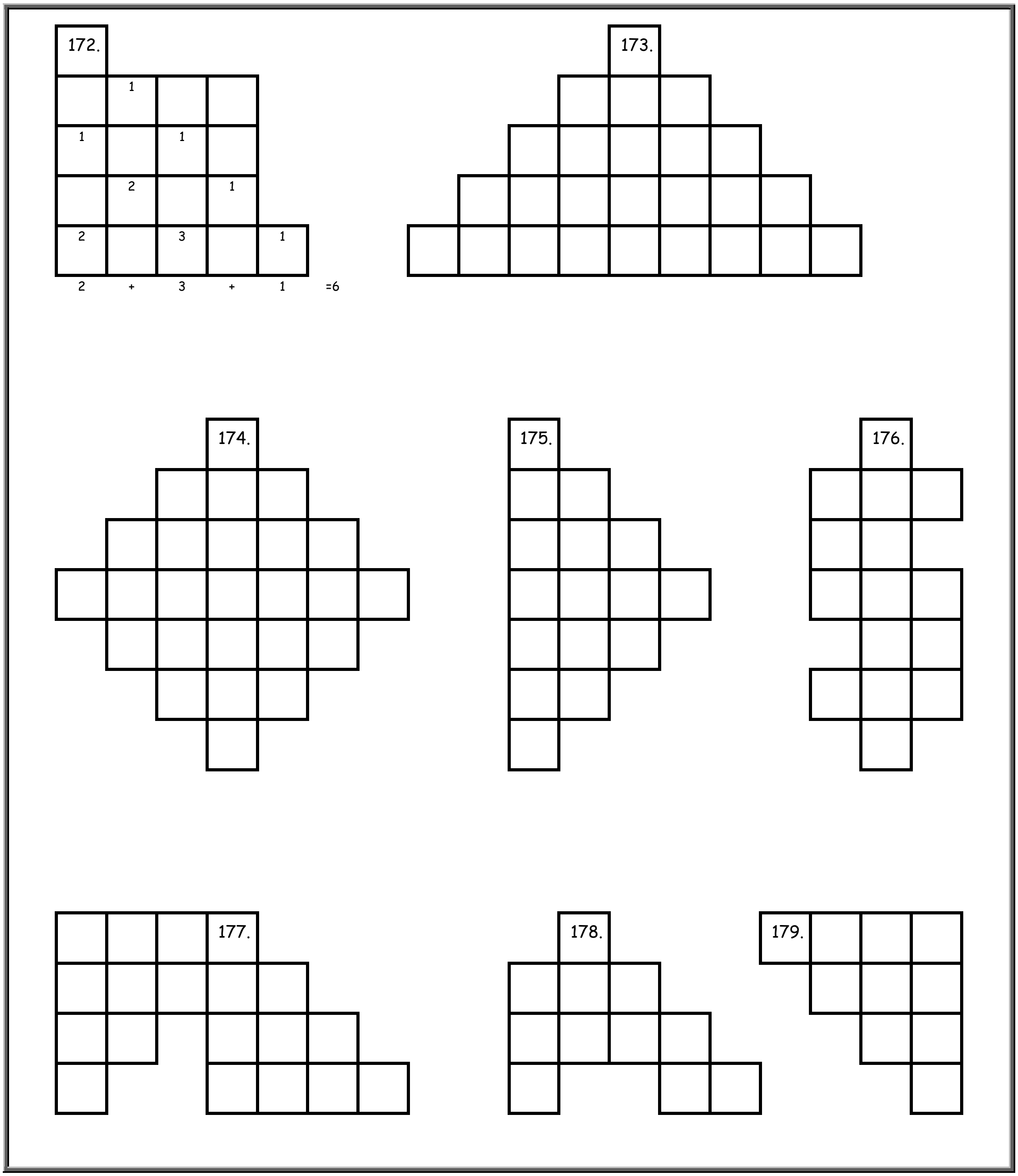


Problem #29: Start inside the box at the number. Move diagonally downward staying inside the boxes, like you would in the game checkers. Determine the number of pathways from the number to the bottom row.



=6

Start inside the box at the number. Move diagonally downward staying inside the boxes, like you would in the game checkers. Determine the number of pathways from the number to the bottom row.



The Binomial Theorem

Problem #30: Expand $(x+y)^0$.

Problem #31: Expand $(x+y)^1$.

Problem #32: Expand $(x+y)^2$.

Problem #33: Expand $(x+y)^3$.

Problem #34: Expand $(x+y)^4$.

Connecting the binomial theorem to Pascal's Triangle.

Compare Pascal's triangle with the coefficients in the binomial expansions.

1	Row 1	$(x+y)^0$	**1**
1 1	Row 2	$(x+y)^1$	$1x+1y$
1 2 1	Row 3	$(x+y)^2$	$1x^2+2xy+1y^2$
1 3 3 1	Row 4	$(x+y)^3$	$1x^3+3x^2y+3xy^2+1y^3$
1 4 6 4 1	Row 5	$(x+y)^4$	$1x^4+4x^3y+6x^2y^2+4xy^3+1y^4$
1 5 10 10 5 1	Row 6	$(x+y)^5$	$1x^5+5x^4y+10x^3y^2+10x^2y^3+5xy^4+1y^5$

Answer the following:

180. Does row 3 have a middle term?
181. Which rows have a middle term?
182. Does $(x+y)^5$ have a middle term?
183. What do you know about A if $(x+y)^A$ has a middle term?
184. How many terms does $(x+y)^5$ have?
185. How many terms does $(x+y)^{500}$ have?
186. How many terms does $(x+y)^A$ have?
187. How many terms does the 4^{th} row have?
188. How many terms does the 4000^{th} row have?
189. How many terms does the N^{th} row have?

How do the x and y exponents change in each term of the binomial expansions?

Row	Binomial	**Binomial expansion**	X exponents	Y exponents
Row 5	$(x+y)^4$	$1X^4+4x^3y+6x^2y^2+4xy^3+1y^4$	190.	191.
Row 6	$(x+y)^5$	$1X^5+5x^4y+10x^3y^2+10x^2y^3+5xy^4+1y^5$	192.	193.

Compare ${}_nC_r$ in each row with the coefficients in the binomial expansions.

${}_0C_0$	Row 1	$(x+y)^0$	**1**
${}_1C_0$ ${}_1C_1$	Row2	$(x+y)^1$	**1**x+**1**y
${}_2C_0$ ${}_2C_1$ ${}_2C_2$	Row3	$(x+y)^2$	$\mathbf{1}x^2+\mathbf{2}xy+\mathbf{1}y^2$
${}_3C_0$ ${}_3C_1$ ${}_3C_2$ ${}_3C_3$	Row4	$(x+y)^3$	$\mathbf{1}x^3+\mathbf{3}x^2y+\mathbf{3}xy^2+\mathbf{1}y^3$
${}_4C_0$ ${}_4C_1$ ${}_4C_2$ ${}_4C_3$ ${}_4C_4$	Row5	$(x+y)^4$	$\mathbf{1}x^4+\mathbf{4}x^3y+\mathbf{6}x^2y^2+\mathbf{4}xy^3+\mathbf{1}y^4$
${}_5C_0$ ${}_5C_1$ ${}_5C_2$ ${}_5C_3$ ${}_5C_4$ ${}_5C_5$	Row6	$(x+y)^5$	$\mathbf{1}x^5+\mathbf{5}x^4y+\mathbf{10}x^3y^2+\mathbf{10}x^2y^3+\mathbf{5}xy^4+\mathbf{1}y^5$

Observation #1: Study Pascal's triangle again. What does the r value in ${}_nC_r$ correspond to: the x-exponent or the y-exponent?

Observation #2: In the expansion of $(x+y)^4$ what do the exponents add to in every term?

Observation #3: In the expansion of $(x+y)^5$ what do the exponents add to in every term?

Observation #4: In the expansions above what happens to the x-exponents as we read left to right?

Observation #5: In the expansions above what happens to the y-exponents as we read left to right?

Observation #6: What does ${}_nC_r$ help us find in each term of the above binomial expansions?

Problem #35: Expand $(x+y)^6$ using what you have learned from Pascal's triangle.

- Find the coefficients of each term.
- Find the combinations of x and y for each term.

Expand $(X+Y)^6$ using the Binomial Theorem

194. How many terms will there be?

- The coefficient of each term can be found using ${}_6C_{\underline{0}} \rightarrow {}_6C_{\underline{6}}$
- X exponents start at 6 and decrease by 1 until it reaches 0
- Y exponents start at **0** and increase by 1 until it reaches 6
- Each term is the product of the above: ${}_nC_r x^{n-r} y^r$

Term 1		Term 2		Term 3		Term 4		Term 5		Term 6		Term 7	
${}_6C_0$	= 1	${}_6C_1$	= 6	${}_6C_2$	= 15	${}_6C_3$	= 20	${}_6C_4$	= 15	${}_6C_5$	= 6	${}_6C_6$	= 1
X^6	$= X^6$	X^5	$= X^5$	X^4	$= X^4$	X^3	$= X^3$	X^2	$= X^2$	X^1	$= X^1$	X^0	= 1
Y^0	= 1	Y^1	= Y	Y^2	$= Y^2$	Y^3	$= Y^3$	Y^4	$= Y^4$	Y^5	$= Y^5$	Y^6	$= Y^6$
X^6		$+ 6X^5Y$		$+ 15X^4Y^2$		$+ 20X^3Y^3$		$+ 15X^2Y^4$		$+ 6XY^5$		$+ Y^6$	

$$(X+Y)^6 = X^6 + 6X^5Y + 15X^4Y^2 + 20X^3Y^3 + 15X^2Y^4 + 6XY^5 + Y^6$$

195. Expand $(m-2)^4$ using the binomial theorem. (Pay attention to the (-2))

Term 1	Term 2	Term 3	Term 4	

$m^4-8m^3+24m^2-32m+16$

196. Expand $(2m-1)^4$ using the binomial theorem.

$16m^4-32m^3+24m^2-8m+1$

197. Expand $(2m-3)^4$ using the binomial theorem.

198. Expand $\left(m-\frac{1}{2}\right)^5$ using the binomial theorem.

$m^5-\frac{5}{2}m^4+\frac{5}{2}m^3-\frac{5}{4}m^2+\frac{5}{16}m-\frac{1}{32}$

199. Expand $(m-2)^5$ using the binomial theorem.

200. Expand $(3m-1)^5$ using the binomial theorem.

201. Find the first 4 terms of the binomial expansion $(m-1)^{15}$.

202. Find the first 4 terms of the binomial expansion $(m-2)^{8}$.

`

203. Find the last 3 terms of the binomial expansion $\left(\frac{1}{2}m-2\right)^{9}$.

204. Find the last 3 terms of the binomial expansion $(2m-n)^{10}$. OMIT (Extra practice only

Problem #36: Find the 5^{th} term of the binomial expansion $(X\text{-}Y)^{10}$.

The Binomial Expansion formula

	Term 1	Term 2	Term 3		Term (k+1)		Term (n+1)
$(X+Y)^n=$	${}_nC_0X^nY^0+$	${}_nC_1X^{n-1}Y+$	${}_nC_2X^{n-2}Y^2+$	...+	${}_nC_kX^{n-k}Y^k+$	...+	${}_nC_nX^0Y^n$
$(X+Y)^6=$	${}_6C_0X^6+$	${}_6C_1X^5Y+$	${}_6C_2X^4Y^2+$	...+	...+	...+	${}_6C_6Y^6$

Find the (k+1)th term $t_{k+1}={}_nC_kX^{n-k}y^k$ This formula is given on the provincial exam	Find the Kth term $t_k={}_nC_{k-1}X^{n-(k-1)}y^{k-1}$ This formula is not given on the provincial exam

205. Find the 5th term of the binomial expansion $(X-Y)^{10}$.

By formula	By hand
$t_{k+1}={}_nC_kX^{n-k}y^k$ Remember $t_5=t_{4+1}$ so $t_{4+1}={}_{10}C_4X^{10-4}(-Y)^4$ $t_{4+1}={}_{10}C_4X^6Y^4$ $t_{4+1}=210X^6Y^4$	Term1→Term2→Term3→Term4→Term5 ${}_{10}C_0\to{}_{10}C_1\to{}_{10}C_2\to{}_{10}C_3\to{}_{10}C_{\underline{4}}$ Remember that the 4 is the y exponent ${}_{10}C_4$ X^{10-4} Y^4 ${}_{10}C_4X^6Y^4=210\ X^6Y^4$

206. Find the 8th term of the binomial expansion $(X-Y)^{10}$

By formula	By hand
$t_{k+1}={}_nC_kX^{n-k}y^k$ Remember $t_8=t_{7+1}$ so $t_{7+1}={}_{10}C_7X^{10-7}(-Y)^7$ $t_{7+1}=-{}_{10}C_7X^3Y^7$ $t_{7+1}=-210X^3Y^7$	Term8←Term9←Term10←Term11←Start ${}_{10}C_7\leftarrow{}_{10}C_8\leftarrow{}_{10}C_9\leftarrow{}_{10}C_{10}\leftarrow$ Start Remember that the 7 is the y exponent ${}_{10}C_7$ X^3 $(-Y)^7$ ${}_{10}C_7X^3(-Y)^7=-120X^3Y^7$

Find the following term of the binomial expansion.

207. $(m-2)^8$ Find the 3rd term.

$112m^6$

208. $(2m-n)^{10}$ Find the 9th term.

209. $(m-1)^{15}$ Find the 12th term.

	Term 1	Term 2	Term 3		Term (k+1)		Term (n+1)
$(X+Y)^n=$	${}_nC_0X^nY^0+$	${}_nC_1X^{n-1}Y+$	${}_nC_2X^{n-2}Y^2+$	...+	${}_nC_kX^{n-k}Y^k+$	...+	${}_nC_nX^0Y^n$
$(X+Y)^6=$	${}_6C_0X^6+$	${}_6C_1X^5Y+$	${}_6C_2X^4Y^2+$	...+	${}_6C_4X^2Y^4+$	...+	${}_6C_6Y^6$

Find the middle term of the binomial expansion.

210. Find the middle term of the binomial expansion, $(m-2)^8$.

$1120m^4$,term 5 is the middle term

211. Find the middle term of the binomial expansion, $(2m-n)^{10}$.

212. Find the middle term of the binomial expansion, $(m-1)^{16}$.

Given a term from a binomial expansion, determine the following:

213. What term in the expansion would $15X^4Y^2$ be?

214. How many terms would there be?

215. What term in the expansion would $36X^5\,Y^6$ be?

216. How many terms would there be?

217. What term in the expansion would $40X^{20}\,Y^4$ be?

218. How many terms would there be?

219. What term in the expansion would $40X^{50}\,Y^4$ be?

220. How many terms would there be?

221. How many terms would there be in a binomial expansion if $200X^5Y^a$ is a term?

222. What term in the expansion would $36X^aY^b$ be?

223. How many terms would there be in a binomial expansion if $40X^{a-1}Y^{b+1}$ is a term?

224. What term in the expansion would $40X^{a+1}Y^{b-2}$ be?

Problem #37: In the expansion $(3A-1)^5$, determine the coefficient of the term containing A^2.

225. In the expansion $(3A-1)^5$, determine the coefficient of the term containing A^2.

Remember
$_5C_3(3A)^2(-1)^3$
$=10(9A^2)(-1)$
$=-90A^2$
The coefficient=-90

226. In the expansion $(2A-3)^8$, determine the coefficient of the term containing A^6.

227. In the expansion $(3A-1)^{13}$, determine the coefficient of the term containing A^2.

228. Find X if XA^5B^3 is one of the terms of the expansion $(A+B)^8$.

229. Find X if XA^5B is one of the terms of the expansion $(A+B)^6$.

230. Find X if XA^3 is one of the terms of the expansion $(3A-1)^5$.

Review of Pascal's Triangle and the Binomial Theorem

231. What term in the expansion would $200X^{20}Y^{30}$ be?

232. Does $(A+B)^{120}$ have a middle term?

233. Find $_nC_r$ for position 7 in the 13th row of Pascal's triangle.

234. Does row 7 of Pascal's triangle have a middle term?

235. What term in the expansion would $230X^{11}Y^{39}$ be?

236. How many terms does the 39th row of Pascal's triangle have?

237. $_{49}C_9 + {}_{49}C_{10}=$

238. How many terms does $(A+B)^{11}$

239. Which $_nC_r$ is equivalent to $_{60}C_1$?

240. How many terms does $(A+B)^8$

241. Does $(A+B)^{67}$ have a middle term?

242. Does row 14 of Pascal's triangle have a middle term?

243. How many terms does the 90th row of Pascal's triangle have?

244. What term in the expansion would $23X^{21}Y^3$ be?

245. Find $_nC_r$ for position 4 in the 71st row of Pascal's triangle.

246. omit

247. omit

248. omit

Unit Review Questions.

249. There are 10 different books. How many different ways can 4 books be placed on a shelf?

250. A true false test has 5 questions. How many different answer keys are possible?

251. Given: 10 question True False test Answer key has 3T and 7F. How many different answer keys are possible?

252. The Final Score in a Baseball game was 7-3. How many possible 5 inning scores are there?

253. Given: 10 question multiple-choice. Answer key has 4 A's, 3B's, 2C's and 1D. How many answer keys are possible?

254. There are 10 different balls. How many different ways can you select 4 balls and put them in a backpack?

255. There are 7 people in a race. How many different ways are there to assign 1^{st}, 2^{nd} and 3^{rd}.

256. How many 4 question exams can be formed by a test bank containing 10 different questions?

257. You have 9 red cards and 6 black cards. You have to discard 11 cards. How many 4-card hands are possible if you keep 2 red, 2 black?

Answers

1. 12	2. 24	3. 90	4. 45	5. 81
6. 900	7. 450	8. 648	9. 75	10. 200
11. 260	12. 8	13. 0.125	14. 32	15. 0.03125
16. 256	17. 0.39%	18. 1024	19. 0.0977%	20. 9765625
21. 0.0000102%	22. 15	23. 21	24. 12	25. 720
26. 24	27. 6	28. 120	29. 5040	30. 40320
31. 362880	32. 720	33. 240	34. 7257600	35. 12
36. 210	37. 210	38. 720	39. 970200	40. 116280
41. 117600	42. 970200	43. 72	44. 117600	45. 9900
46. n^3-3n^2+2n	47. $n+1$	48. 110	49. 504	50. 999000
51. 10200	52. n^2-n	53. n^2+3n+2	54. n^2-2n	55. $\frac{1}{4}(n^2-3n+2)$
56. 11	57. 5	58. 8	59. 2	60. 2
61. 2	62. 30240	63. 336	64. 40	65. 72
66. 15625	67. 117600	68. 24	69. 24	70. 12
71. 45360	72. 140	73. 120	74. 840	75. 21
76. 7560	77. 60060	78. 1680	79. 369600	80. 4200
81. 24	82. 4	83. 210,c	84. 56,c	85. 24360,p
86. 720,p	87. 3 36,p	88. 4060,c	89. 90	90. 30
91. 1900	92. 270	93. 168	94. 1584	95. 22704
96. 329550	97. 31680	98. 700	99. 756	100. 5292
101. 1722	102. 1946	103. 5817	104. 4242	105. 1946
106. 5817	107. 1365	108. 540	109. 66	110. 660
111. 1365	112. 10	113. 11	114. 5	115. 8
116. $\frac{1}{2}\left(n^2-n\right)$	117. $\frac{1}{2}\left(n^2-3n+2\right)$	118. $\frac{1}{2}\left(n^2+n\right)$	119. $\frac{1}{6}\left(n^2+3n+2\right)$	120. 2598960
121. 1287	122. 1287	123. 65780	124. 22308	125. 845000
126. 329550	127. 24	128. 4	129. 211904	130. 103776
131. 171600	132. 845000	133. 108336	134. 1299480	135. 2144480
136. 31	137. 25	138. 6	139. The row number	140. 1
141. The right half	142. The boxes above that touch it	143. ${}_{6}C_{4}$	144. ${}_{17}C_{9}$	145. ${}_{100}C_{19}$
146. ${}_{n}C_{n-10}$	147. ${}_{n+10}C_{n}$	148. ${}_{ab}C_{ab-c}$	149. ${}_{6}C_{2}$	150. ${}_{45}C_{20}$
151. ${}_{a}C_{b}$	152. ${}_{a+1}C_{b+1}$	153. ${}_{a+7}C_{b+2}$	154. ${}_{2a+2}C_{b-4}$	155. ${}_{a-x+1}C_{by}$
156. ${}_{Iluv}C_{math12}$	157. ${}_{5}C_{3}$	158. ${}_{60}C_{0}$	159. ${}_{7}C_{1}$	160. ${}_{100}C_{98}$
161. ${}_{A-1}C_{4}$	162. ${}_{d+2}C_{d-1}$	163. 126	164. 81	165. 60
166. 46	167. 60	168. 30	169. 59	170. 82
171. 90	172. 6	173. 16	174. 20	175. 5
176. 8	177. 5	178. 3	179. 1	180. yes
181. Odd rows	182. no	183. Even exponents	184. 6	185. 501

186. A+1	187. 4	188. 4000	189. N	190. Down by 1
191. Up by 1	192. Down by 1	193. up by 1	194. 7	195. m^4-8m^3+24 m^2-32m+16

196. 16m^4-32m^3+24 m^2-8m+1	197. 16m^4-96m^3+216 m^2-216m+81	198. answered on page
199. m^5-10m^4+40m^3-80m^2+80m-32	200. 243m^5 - 405m^4 + 270m^3 - 90m^2 + 15m^1-1	201. m^{15} - 15m^{14} + 105m^{13} - 455m^{12}
202. m^8 - 16m^7 + 112m^6 - 448m^5	203. -1152m^2, 1152m , -512	204. 180m^2 n^8, -20mn^9, n^{10}
205. ${}_{10}C_4X^6Y^4$	206. ${}_{10}C_7X^3(-Y)^7$	207. 112m^6
208. 180m^2n^8	209. -1365m^4	210. 1120m^4,term 5 is the middle term
211. -8064m^5n^5,term 6 is the middle term	212. 12870m^8, term9 is the middle term	213. 3

214. 7	215. 7	216. 12	217. 5	218. 25	219. 5
220. 55	221. a+6	222. b+1	223. a+b+1	224. b-1	225. -90
226. 16128	227. -702	228. 56	229. 6	230. 270	231. 31
232. yes	233. ${}_{12}C_6$	234. yes	235. 40	236. 39	237. ${}_{50}C_{10}$
238. 12	239. ${}_{60}C_{59}$	240. 9	241. no	242. no	243. 90
244. 4	245. ${}_{70}C_3$	246. ${}_{80}C_{74}$	247. no	248. 25	249. 5040
250. 32	251. 120	252. 32	253. 12600	254. 210	255. 210
256. 210	257. 540	258. 3628800	259.	260.	261.